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Social force model with explicit collision prediction

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Abstract – We introduce a new specification of the social force model in which pedestrians explicitly predict the place and time of the next collision in order to avoid it. This and other specifications of the social force model are calibrated, using genetic algorithms, on a set of pedestrian trajectories, obtained tracking with laser range finders the movement of pedestrians in controlled experiments, and their performance is compared. The results show that the proposed method has a better performance in describing the trajectory set.

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Introduction. – Models of pedestrian behaviour have been proposed starting from the late 1950s [1]. These models were focused on the dynamics of macroscopic quantities (densities and fluxes), and the dynamics of pedestrians was treated in a way similar to that of gases or fluids [2], but later the attention has shifted on a microscopic description, in which the motion of each pedestrian is individually described. These models can be roughly divided into Cellular Automata models using a discrete-space description [3,4], and models using a continuous-space description. Given our interest in introducing human-like collision prediction and avoiding abilities in robots to have them move smoothly in non-overcrowded environments such as shopping malls [5], we are interested in a detailed description of the microscopic motion of pedestrians, and thus we will rely on the latter (agent model) approach.

Local collision avoiding is a basic component of any pedestrian agent model. The calibration of a collision avoiding model has been traditionally done not using local information about individual motion, but data concerning macroscopic collective behaviours; studying phenomena such as pattern formation [6,7], bottleneck oscillations [8] and speed-density relations [9]. The reason of this global approach was related both to the difficulty of tracking individual positions using an automatic system [10] and to theoretical issues with the prediction and modelling of individual human behaviour [11]. Many models have proved to be able to describe qualitatively collective dynamics [12,13], but quantitative information about these behaviours is surprisingly difficult to obtain, in particular in real world situations [14]. Furthermore, we believe that if the pedestrian model is intended to simulate not only macroscopic features of crowd dynamics, but also local motion, calibration should also take in account data regarding individual trajectories.

Lately, with the improvement of automatic tracking of pedestrian trajectories from video and laser sensors [10], a few calibrations on individual trajectories have been performed [15–17], allowing model calibration based on local behaviours, and this work follows this kind of approach. Pedestrian trajectories are tracked using laser range finders [18] in a controlled experiment, that reproduces a situation in which density is relatively low, and pedestrians are moving in many different directions, as we expect to happen in a shopping mall. A general method to calibrate a pedestrian collision avoiding model was introduced in [15], and genetic algorithms have proved to be an effective optimisation tool for this problem [19]. Genetic algorithms also allow the calibration of relatively complex models for which an analytical derivation could be problematic (models using some kind of artificial intelligence, boolean choices etc).

The social force model [20] simulates pedestrian dynamics using interaction forces. It introduces a quite general framework in which the details of the collision avoiding behaviour can be expressed through a function depending on the relative and absolute positions and velocities of the pedestrians. Even in its simplest formulation that uses only information about positions, the model describes correctly many qualitative features of pedestrian behaviour, in particular in the high-density regime. Improved specifications of the model have been introduced, taking into account pedestrian velocities [16].

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Some of these specifications are calibrated in this work, and compared to a new specification that uses a collision avoiding model based on explicit prediction of future collisions. We show that the proposed specification better describes human trajectories with respect to the previous ones, and by a comparison between different specifications, we also discuss some of the features that a collision avoiding model needs to describe pedestrian behaviour, at least at the low-density regime.

**Social force model.** – According to the social force model of pedestrian motion, each pedestrian tries to move in a desired direction \(\mathbf{e}_i\) with a desired speed \(v_i^0\) (i.e., with desired velocity \(\mathbf{v}_i^0 = v_i^0 \mathbf{e}_i\)). The direction of the desired velocity is given by a vector pointing from the present position of the pedestrian \(\mathbf{r}_i\) to her next (sub)goal \(\mathbf{g}_i\), while the speed is the one at which the pedestrian feels more comfortable to walk. The acceleration of the pedestrian is given by a social force term plus a fluctuation term

\[
\frac{d\mathbf{v}_i(t)}{dt} = \mathbf{f}_i(t) = f_i^s(t) + \xi_i(t).
\]

Assuming that the deviations of pedestrians from the straight path leading to their goal are only due to inter-pedestrian collision avoiding interactions, and that the pedestrian adapts her velocity to the desired one in a relaxation time \(k_i^{-1}, f_i^s\) is given by

\[
f_i^s(t) = k_i(v_i^0 - \mathbf{v}_i(t)) + \sum_{j \neq i} f_{i,j}(t),
\]

where \(f_{i,j}\) is the interaction force with pedestrian \(j\).

**Circular specification (CS).** The circular specification [20] assumes forces to depend only on the distance \(d_{i,j}\) between pedestrians. This assumption works quite well under high-density conditions as those occurring during escape situations. The interaction force is

\[
f_{i,j}(t) = Ae^{(d_{i,j}(t))/B} \frac{d_{i,j}(t)}{d_{i,j}(t)},
\]

where \(A\) and \(B\) denote, respectively, the strength and range of the interaction force, \(d\) is the sum of the “radii” of the two pedestrians and \(d_{i,j} \equiv |\mathbf{r}_i - \mathbf{r}_j|\).

**Elliptical specification II (ES2).** Another elliptical specification was proposed in [16]. Now \(b_{i,j}\) in (4) is defined by

\[
2b_{i,j} = \sqrt{(d_{i,j} + \|\mathbf{v}_j - \mathbf{v}_i\|\tau\|^2 - \|\mathbf{v}_j - \mathbf{v}_i\|^2),
\]

where \(\tau\) is the time of pedestrian stride. By derivation the force is

\[
f_{i,j}(d_{i,j}, \mathbf{v}_j) = Ae^{-b_{i,j}/B} \frac{d_{i,j}}{d_{i,j}} + \|\mathbf{d}_{i,j} - \mathbf{y}_{i,j}\| \times \left(\frac{d_{i,j}}{d_{i,j}} + \frac{\mathbf{d}_{i,j} - \mathbf{y}_{i,j}}{\|\mathbf{d}_{i,j} - \mathbf{y}_{i,j}\|}\right),
\]

where

\[
y_{i,j} = d_{i,j} - \mathbf{v}_j \tau.
\]

This specification takes in account not only the relative distance, but also the movement of the other pedestrian.

**New elliptical specification (NES).** The new elliptical specification was introduced in [19] in order to take in account the absolute velocity, which has an influence in pedestrian head-on interactions, assuming

\[
2b_{i,j} = \sqrt{(d_{i,j} + \|\mathbf{v}_j - \mathbf{v}_i\|\tau\|^2 - \|\mathbf{v}_j - \mathbf{v}_i\|\tau\|^2),
\]

By derivation from the potential 4, we have

\[
f_{i,j}(d_{i,j}, \mathbf{v}_j) = Ae^{-b_{i,j}/B} \frac{d_{i,j}}{d_{i,j}} + \|\mathbf{d}_{i,j} - \mathbf{y}_{i,j}\| \times \left(\frac{d_{i,j}}{d_{i,j}} + \frac{\mathbf{d}_{i,j} - \mathbf{y}_{i,j}}{\|\mathbf{d}_{i,j} - \mathbf{y}_{i,j}\|}\right).
\]

**Collision prediction (CP).** These previous models introduce social forces that reproduce the collision avoiding behaviour of pedestrians, but do not use an explicit computation of possible collisions. The elliptical specifications introduce the parameter \(\tau\) as the time of a pedestrian stride, assuming that the pedestrian will try to have some free space for her motion. We believe that, even if at an unconscious level, pedestrians have the ability to understand the time at which a collision is probably going to happen and to modify their motion in order to avoid it, and that we could substitute \(\tau\) with an explicit computation of this time in order to simplify the model removing the parameter \(\tau\). In [21] a theoretical and experimental analysis of the 1D fundamental diagram and of single lane head-on collisions was performed, showing the necessity to introduce foresight and non-additive interactions in the social force model. Foresight was introduced as a linear extrapolation of future positions at a fixed time \(\tau\), but a full 2D model was not described. A model using a computation of future possible collisions has been introduced in [22]. A similar model has been introduced in [23], and
used for the study of collision avoiding behaviours from an evolutionary point of view [24,25]. In its simplest form, i.e. assuming straight motion in the computation of future collisions, it is not computationally expensive and has been used in large-scale pedestrian simulations [26,27].

The key concept of the model is that each pedestrian will try to compute, for each approaching pedestrian in her environment, the time in the future at which their relative distance will be minimum, assuming straight motion. She will then check for the minimum of these times, and will predict the positions of herself and all the other pedestrians at that point in the future. Interaction forces will then be circular symmetric forces as those used in CS, but based on this future situation, which is assumed to be the most interesting for the pedestrian since it is the first one in which a collision can happen. More in detail, the force \( f_i \) can still be written as in eq. (2), where the interaction force is a function of relative positions and velocities, and absolute velocity. If the angle between \( d_{i,j} \) and \( v_i, v_j \), \( \theta_{i,j} \), is smaller than \( \pi/4 \) we compute the time \( t_{i,j} \) at which the pedestrians \( i \) and \( j \) will be closest, assuming straight motion (see fig. 1) (a similar concept, even if in a quite different model using synthetic vision, is introduced in [28]). If \( \theta_{i,j} > \pi/4 \) then \( t_{i,j} = \infty \). This computation is performed for each pedestrian. We then compute \( t_i = \min_j \{t_{i,j}\} \) and, assuming once again straight motion at constant velocity, the projected future distances at time \( t_i, d'_{i,j}(t) \) for each \( j \) (if \( t_i = +\infty \) no interaction occurs). The contribution of each \( j \) is then obtained as

\[
 f_{i,j}(\{d_{i,j}\}, \{v_{i,j}\}, v_i) = A \frac{v_j}{t_i} e^{-d_{i,j}/B} \frac{d'_{i,j}(t_i)}{d_{i,j}(t_i)}
 \]  

(10)

The term \( v_i/t_i \) introduces a dependence on individual velocity assuming that the pedestrian wants to be able to stop in \( t_i \), and thus for example in case of a frontal collision her breaking strength should be of order \( v_i/t_i \). Some thought experiments show that this kind of interaction prescribes a realistic and efficient collision avoiding behaviour. For example, in the situation of an interaction with almost perpendicular velocities, it leads

the pedestrian who is closer to the crossing point to accelerate, and the other one to decelerate (fig. 2). The terms \( \{d_{i,j}\}, \{v_{i,j}\} \) in (10) recall us that \( t_i \) does not depend only on a single pair \( d_{i,j}, v_i \), but on the whole set \( \{d_{i,j}\}, \{v_{i,j}\} \). Thus this model, in opposition to the usual specifications of the social force model, is not additive: the interaction with a given pedestrian is modified by the presence of the others.

**Difference between the models.** All the models used in this paper take in account relative positions while CS is the only model that does not use any velocity dependent information. ES1 uses the opponent’s velocity to compute interaction forces, while CP, ES2 and NES use relative velocity. We expect these three models to outperform the other models in a low-density environment in which non-trivial collision avoiding behaviours are present, since the relative velocity information is crucial to predict and avoid collisions (see fig. 2). NES and CP use also information coming from the pedestrian’s absolute velocity, which should be useful in describing the head-on behaviour [19], even if we don’t expect this information to be crucial in the studied regime. The main difference between CP and ES2-NES resides in the definition of the “collision time” \( t_i \), i.e. the positions of all other pedestrians are computed at a time interval computed on the run, on the basis of all the other pedestrians’ positions and velocities, while in the other two models the parameter \( \tau \) plays a similar role (at least according to our interpretation), but it is fixed for all interactions. The use of \( t_i \) also causes CP to be non-additive (a detailed description of this mechanism can be found in [24]). The computational cost is very similar in all the models. In our implementations, that have no claim to be optimal, CP resulted to be slightly slower than CS and slightly quicker than elliptical specifications. During
calibration, the simulation of $1.12 \cdot 10^8$ trajectories was performed in 2299s using CS, 2689s using CP and 3590s using NES.

Anisotropy. Anisotropy depending on the angle $\phi_{i,j}$ between $v_i$ and $d_{i,j}$ is introduced in all the models weighting the interaction force by a factor
\[
w(\phi_{i,j}) = \left( \lambda + (1 - \lambda) \frac{\sin(\phi_{i,j})}{2} \right),
\]
\[0 \leq \lambda \leq 1,
\]
as is usually done in the social force model [16].

Parameters. All the parameters in the model are obtained by calibration using experimental trajectories. In order not to increase too much the parameter space, only the preferred velocity $v^0$ will be considered as pedestrian dependent, while the other parameters will be considered as common to all pedestrians. For CP, the parameters are $v^0$, $A$, $B$, $k$, and $\lambda$. CS uses also the parameter $d$, while elliptical models introduce the parameter $\tau$.

Integration. The integration of the system of differential equations defined by (1) will be performed using an Euler method, (whose time step will be $\Delta t = 0.2$ seconds, i.e. the regular time step at which experimental pedestrian trajectories are tracked). The use of an Euler integrator allows us to interpret eq. (1) as the “cognitive dynamics” of the system. Following this interpretation, at each discrete time step, each pedestrian decides to modify her velocity according to eq. (1) (in an impulsive way, i.e. the velocity is suddenly modified and maintained until the next interaction). If a physical dynamics is introduced in the system, its time scale should be $\Delta t' \ll \Delta t$. Later we will show how we can introduce a simplified physical dynamics that can be solved exactly in continuous time.

Calibration. – Experiments. For calibration we use a set of pedestrian trajectories obtained in a controlled experiment performed in our laboratory, in which 8 subjects took part. Each subject was given a start and goal point, and was prescribed to walk as naturally as possible towards the goal. The trajectories of pedestrians were tracked using laser range finders [18] in a square area with an 8 meters side (start and goal points were located outside the area, but trajectories were tracked only when the pedestrians were crossing the area). The tracked trajectories were smoothed using a moving average approach obtaining trajectories with a regular 0.2 second time step. A total of 224 trajectories were obtained, 96 in an experimental condition (C1) in which 4 pedestrians were simultaneously present in the area, 128 in a condition (C2) with 8 pedestrians. During these experiments, the density in the area is around 0.1 persons per squared meter, which corresponds to the possibility of walking with a very comfortable speed [14]. Nevertheless, start and goal points were chosen in order to put pedestrians in a condition in which they have to solve a non-trivial collision avoiding problem, avoiding other pedestrians coming from different directions, but they can do it without constraints due to severe physical density, which is the situation we are interested to study in this work.

Calibration method. In the following we are going to introduce a method based on genetic algorithms to calibrate any collision avoiding model given a set of trajectories that are supposed to be completely describable using such a model. The model will be treated as a black box that, given a set of parameters and the state of the system (current positions and velocities of pedestrians) gives the state at the next step (next-step positions and velocities), and will be defined by the whole set of parameters of the pedestrians. In order to make this evolution deterministic and thus more easily testable we will remove the stochastic term in eq. (1).

The solutions are evaluated on the basis of the similarity of experimental trajectories and simulated trajectories obtained using the same initial conditions. We will test both the ability of the model to reproduce the behaviour of a single pedestrian when the positions and velocities of the others are given from the data, and its ability to simulate all the pedestrians simultaneously. The second approach allows us to introduce physical constraints, that lead to a more effective collision avoiding behaviour that can be safely used in simulations and practical applications.

Our fitness function has thus two terms. The first one is more or less equivalent to the approach used in [15]. Our experimental knowledge of the system is given by the set of trajectories $\{r_i(t_k)\}$, where $i$ is an index running over all the pedestrians recorded in the experiment, and $k$ over all the time steps at which these pedestrians have been tracked. For each $i$ we obtain the simulated trajectory $r'_i\prime$ imposing the initial condition $r'_i(t_0) = r_i(t_0)$, keeping fixed the positions and velocities of the other pedestrians. In other words, $r'_i\prime$ is obtained by having a single virtual pedestrian interacting with the real pedestrians. The first term of the fitness function is
\[
\mathcal{F}_1 = \sum_{i,k} \frac{\| r_i(t_k) - r'_i(t_k) \|}{N_D},
\]
where $N_D = \sum_{i,k} 1$ is the number of points in the data set. The meaning of this term of the fitness function is quite clear, since by its optimisation we are trying to have simulated pedestrian to behave as similar as possible to actual pedestrians given the same environmental conditions. We have nevertheless verified that using this approach, the optimisation process leads to collision avoiding behaviours that are weaker than those of actual pedestrians, i.e. after calibration simulated pedestrians pass closer to each other than actual pedestrians do. We checked the shortest distance attained by pedestrians during our experiments, that resulted to be $2R = 0.6$ meters, and introduced a physical constraint to avoid pedestrians to reach a shorter distance during simulations. We thus defined a second fitness term, $\mathcal{F}_2$, in the following way. We introduce a physical dynamics, assuming that pedestrians
can be described as 2D discs with radius $R$ undergoing elastic collisions. This is obviously not a realistic physical dynamics for pedestrians, but can be solved in continuous time [24] without an high computational cost, and thus it is an efficient way to introduce physical constraints. Furthermore, it provides a rigorous measure for collisions between pedestrians, as the amount of physical momentum exchanged in collisions. We will name $r_{ij}$ a trajectory obtained using this physical dynamics, in an environment in which all the pedestrians are simultaneously simulated. The second term of the fitness function is thus

$$\mathcal{F}_2 = -\left( \beta P + \sum_{i,k} \| r_i(t_k) - r_i'(t_k) \| \right) / N_D, \quad (13)$$

where $P = \sum_{\text{collisions}} |p|$ is the total amount of momentum exchanged in collisions. Introducing the constant $\beta = 1$ (dimensionally time over mass, even if all masses are assumed equal to 1 in the system) the system will minimise the fictitious effect of physical collision in dynamics.

Assigning the same weight to the two terms the fitness function will be

$$\mathcal{F} = (\mathcal{F}_1 + \mathcal{F}_2)/2 \quad (14)$$

measuring the average distance of simulated and actual trajectories under the two conditions.

Genetic algorithm. To calibrate and evaluate the models and the roughness of their fitness landscape, we use multiple GA runs. This is not necessarily the computationally optimal approach and other algorithms, as TD learning, could be attempted in future. At the first generation each gene, coded as a real (double precision floating point) number, is randomly chosen using a uniform distribution in a reasonable range, obtaining 500 different genomes. Each genome in a new generation is obtained by crossover of two genomes selected using tournament selection (5 genomes are randomly chosen, and the one with highest fitness is selected), and then mutated with Gaussian error and probability 0.1. For each calibration test we performed 50 different runs, each run lasting 1000 generations.

Results and discussion. – We calibrated models on all trajectories. We also tested the model calibrated on C1 to see how it works on C2 trajectories. The results for average (and standard deviation) of best solutions over all the GA runs, and best overall solutions are shown in tables 1 and 2. These results show a clear performance hierarchy between models, with CP having the maximum fitness, followed by NES-ES2, ES1 and CS. The average fitness evolution for calibration over all trajectories is shown in fig. 3, while the average fitness evolution for solutions calibrated on C1 and tested on C2 is shown in fig. 4. The values of calibrated parameters (best solution, calibrated on both conditions) are shown in table 3.
Our results show that introducing an explicit computation of collisions in the social force model improves the capability of the model to reproduce pedestrian trajectories. The differences between CP and ES2-NES were statistically significant under all conditions, having CP outperforming the other two models around 5% on trajectories with four pedestrians, and around 15% on trajectories with 8 pedestrians or on all trajectories. When tested after calibration, CP outperformed the other models around 15%. The difference between the models is due probably to the fact that both CP and ES2-NES take in consideration the future situation, but while ES2-NES do it at a fixed time step $\tau$, CP explicitly computes the time of the probable collision. Furthermore CP does it in a non-additive way, introducing some kind of priority in the interaction between different neighboring pedestrians. The new model seems particularly successful in describing the more complex situation in which 8 pedestrians are present. The value of $\tau \approx 1.7$s for ES2-NES suggests that this parameter, more than for the length of the streets, accounts for the time to a next collision (interaction with a pedestrian located at a few meters of distance) and thus an explicit introduction of this computation seems reasonable. The fitness mean square deviation always assumes the minimum value in CP under all calibration conditions, suggesting that the reduction of the number of parameters resulted in a smoother fitness landscape and that CP needs less to be “fine tuned” than the other models. No significant difference is found between ES and NES, as predictable since in a situation in which the density is quite low and pedestrians have different starting and goal points there should be no need to describe the head-on behaviour of pedestrians walking in the same direction. Nevertheless these two models significantly outperform ES1 and even more CS, showing the need to take in account the future position of pedestrians in computing the interaction forces, in particular in the low-density regime. These models were particularly poor when dealing with physical constraint ($F_1 = -0.380$ and $F_2 = -0.533$ for CS best solution in contrast to $F_1 = -0.269$ and $F_2 = -0.309$ for CP) showing their difficulty in reproducing actual trajectories while avoiding collisions.

Conclusions. – We believe that our results show that an explicit computation of collision times improves the description of local pedestrian motion, allowing for a precision smaller than the size of the human body, at least on the small scale considered. Furthermore the proposed model has less parameters to be calibrated than the previous models, substituting the constant $\tau$ with the computed collision time. Our results also show the necessity to take in account relative velocity of pedestrians in low-density collision avoiding. Future developments regard the study of the performance of the proposed model at higher densities, and the introduction of a non-circular interaction force that takes in account the actual shape of the human body.

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